in $|0\rangle$. Similarly, if the first qubit is found in $|1\rangle$, the second qubit will also be in $|1\rangle$. This correlation holds regardless of the distance between the qubits and enables a level of coordination that classical bits cannot achieve. This means the probability of observing the states $|01\rangle$ or $|10\rangle$ is 0.

Applications of entanglement are widespread in quantum algorithms. For example, quantum entanglement enables superdense coding, a quantum algorithm that aims to transmit a number of classical bits of information using fewer qubits. Another application of entanglement is quantum teleportation. This theoretical process allows for information transfer through entangled particles. In this process, two parties—located at any distance from each other—use a shared entangled state to transfer information about a given quantum state from one location to another. This process is referred to as the teleportation of a quantum state.

In 2012, researchers achieved quantum teleportation over a distance of 143 km, from La Palma to Tenerife *arXiv:1205.3909 [quant-ph]*.

Current Quantum Technology

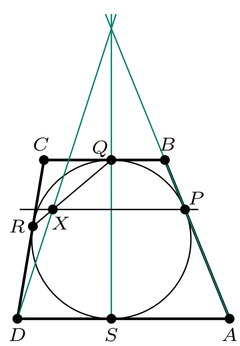
Quantum technology is still in its early stages, but there has been significant progress. Large tech companies like Google, IBM, and Intel are building computers with increasingly more qubits; however, they are struggling to reduce error rates. Currently, we have noisy intermediate-scale quantum (NISQ) devices, which have enough qubits to perform certain quantum computations but are prone to errors and decoherence (the loss of quantum information). Scientists are working on developing error-correction techniques and „quantum error-correcting codes" to reduce decoherence. The ultimate goal is to build error-resistant quantum computers with applications across various industries. Large-scale quantum computers may still be a decade away, but the current pace of innovation suggests a bright future for quantum technology.

The world's largest quantum computer is a 1,180-qubit system developed by Atom Computing. Each qubit is a neutral atom that is trapped and controlled by an array of lasers.

Problems

Edited by Dominik BUREK

M 1813. Let $ABCD$ be a trapezoid ($DA \parallel CB$) circumscribed on a circle that is tangent to the sides AB, BC, CD, and AD at points P, Q, R, and S, respectively. A line passing through P and parallel to the bases of the trapezoid intersects the line QR at point X. Prove that the lines AB, QS, and DX intersect at a single point.



M 1814. Given numbers a, $b > 1$, for which

$$a + \frac{1}{a^2} \geqslant 5b - \frac{3}{b^2}.$$

Prove that

$$a > 5b - \frac{4}{b^2}.$$

M 1815. Consider two integers $n > 20$ and $k > 1$ such that $k^2|n$. Prove that there exist positive integers a, b, and c such that

$$n = ab + bc + ca.$$

Edited by Andrzej MAJHOFER

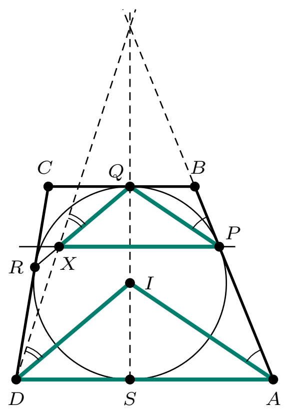
F 1117. In a tightly sealed cylinder, under the piston, there are $m = 10$ g of liquid water. A very rapid movement of the piston causes the pressure in the cylinder to drop to nearly zero. The surrounding temperature and the cylinder with water are at $0°C$. How much ice will be produced as a result of this process? It can be assumed that initially, under the piston, there was only liquid water. The latent heat of fusion of water is $L_f \approx 334$ J/g, and the latent heat of vaporization is $L_v \approx 2260$ J/g.

F 1118. In a sealed container, there is a mixture of helium and neon. The mixture is in thermodynamic equilibrium, with the number of moles of neon and helium being the same. A very small hole is made in the wall of the container. What will be the composition of the gas beam escaping from the container just after the hole is made? In atomic mass units, the atomic masses are: helium $\mu_{\mathrm{He}} = 4$, and neon $\mu_{\mathrm{Ne}} = 20$.

Solutions on page 24

Solutions to the problems from page 9

Solutions to the problem M 1813.



Let I be the center of the inscribed circle in the considered trapezoid. Then

$$\sphericalangle IAP = \tfrac{1}{2}\sphericalangle DAB = \tfrac{1}{2}(180^\circ - \sphericalangle ABC) = \sphericalangle QPB,$$

hence $PQ \parallel AI$. Similarly, $DI \parallel QR$, so the triangles ADI and PXQ have corresponding sides parallel, and therefore they are homothetic. The center of this homothety is the point of intersection of the lines AB, QS, and DX.

Solutions to the problem M 1814.

Assume the opposite, i.e.,

$$a \leqslant 5b - \frac{4}{b^2}.$$

Then

$$5b - \frac{4}{b^2} + \frac{1}{a^2} \geqslant a + \frac{1}{a^2} \geqslant 5b - \frac{3}{b^2},$$

hence

$$\frac{1}{a^2} \geqslant \frac{1}{b^2},$$

so $a \leqslant b$. However, then

$$a + \frac{1}{a^2} \geqslant 5b - \frac{3}{b^2} \geqslant 5a - \frac{3}{a^2},$$

which means that

$$\frac{4}{a^2} \geqslant 4a,$$

so $a \leqslant 1$ – a contradiction.

Solutions to the problem M 1815.

We want to show that there exist positive integers a, b, and c such that

$$n + a^2 = (a+b)(a+c),$$

so it is enough for the number $n + a^2$ to be a product of two integers greater than a. Based on the assumptions of the problem, we can find such a prime number p and an integer $m > 0$ such that $n = p^2 m$. Consider four cases:

1) $m + 1 > p$. In this case, we can take $a = p$, since $n + a^2 = p^2 \cdot (m+1)$ and both p^2 and $m + 1$ are greater than a.
2) $m + 1 < p$ and $m + 1$ is a composite number. Let $m + 1 = st$ for some integers $s, t > 1$. Again, we can take $a = p$, since $n + a^2 = ps \cdot pt$ and both ps and pt are greater than a.
3) $m + 1 < p$ and $m + 1$ is a prime number. Let $m + 1 = q$ and divide p by q with remainder: $p = \ell q + r$, where $r > 0$. Take $a = r$, then $n + a^2 = q \cdot (\ell^2 mq + 2\ell mr + r^2)$ and both factors are greater than r.
4) $m + 1 = p$. Then obviously $n = p^3 - p^2 > 20$, so we can assume $p \geqslant 4$. We have
$$n + 6^2 = (p+3) \cdot (p^2 - 4p + 12),$$
where both factors on the right-hand side are greater than 6.

In each of these cases, we obtained the desired factorization, so the theorem has been proven.

Solutions to the problem F 1117.

After the rapid decrease in pressure, the water begins to evaporate throughout its entire volume. The resulting vapor absorbs heat from the liquid water, causing it to freeze (the water has a temperature of $0°C$). The evaporation process is very fast, so the mass ratios of ice and vapor right after the pressure drop are determined only by the values of latent heat of vaporization and fusion. Let m_l denote the mass of ice produced. We have:

$$L_f \cdot m_l = L_v \cdot (m - m_l).$$

Solving for the mass of ice:

$$m_l = \frac{L_v m}{L_v + L_f}.$$

Numerically, $m_l \approx 8.71$ g. In equilibrium, which will be achieved as a result of further slow sublimation/resublimation processes, the mass of ice will also depend on the volume under the piston available for the water vapor.

Solutions to the problem F 1118.

In thermodynamic equilibrium, the average kinetic energies of helium and neon atoms are equal and proportional to the temperature in Kelvin. This means that their average velocities are inversely proportional to the square roots of their masses:

$$\frac{v_{\mathrm{He}}}{v_{\mathrm{Ne}}} = \sqrt{\frac{m_{\mathrm{Ne}}}{m_{\mathrm{He}}}} = \sqrt{5}.$$

If only one type of gas were present in the container, the time to empty the container of helium would be $\sqrt{5}$ times shorter than the time to empty it of neon. Since both gases can be treated as ideal gases, and the atoms do not interact with each other, the number of helium atoms escaping from the container per unit time will be $\sqrt{5}$ times greater than the number of neon atoms:

$$\frac{n_{\mathrm{He}}}{n_{\mathrm{Ne}}} = \sqrt{5}.$$

As the gas escapes, the composition of the beam will change because helium escapes faster than neon.