Club 44 F

Solution submission deadline: 30 VI 2025

Physics Problems No. 796, 797

Edited by Elżbieta ZAWISTOWSKA

796. In the circuit shown in Figure 1, the switch was closed to position 1, and after the currents stabilized, it was quickly switched to position 2. Assuming that the inductors L_1 and L_2 are ideal, determine the amount of heat dissipated in the resistor R after switching. The electromotive force of the source is $\mathcal{E}$, and its internal resistance is r.

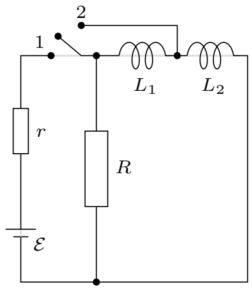

Fig. 1

797. At the center of the bottom of a rectangular barge with a length of $a = 80$ m, a width of $b = 10$ m, and a height of $c = 5$ m, a hole with a diameter of $d = 1$ cm has formed. Estimate the time after which the barge will sink if no water is pumped out. The barge is open at the top, carries no cargo, and the initial height of the hull above the water level is $h = 3{,}75$ m.

Solutions to problems from issue 12/2024

We recall the problem statements:

788. A system of blocks is placed on a frictionless table, as shown in Figure 2. The coefficient of friction between the blocks of masses M and m is μ. The blocks of masses m are connected by a massless, inextensible string. The lower right block is pulled parallel to the table with a force F. Find the accelerations of all blocks.

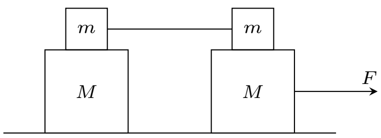

Fig. 2

789. A hexagonal pencil is pushed along a horizontal surface, as shown in Figure 3. What must be the coefficient of friction μ between the pencil and the surface for the pencil to slide without rotating?

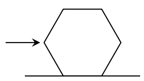

Fig. 3

788. The accelerations of the blocks of mass m are the same because they are connected by an inextensible string. The friction between blocks 3 and 4 (Fig. 4) is static friction. If block 3 were to slide on block 4, the friction force between them, $T_4 = \mu mg$, would be smaller than the tension force N, causing block 2 to move left, which is impossible. Therefore, the accelerations of blocks 2, 3, and 4 are equal. Let us denote them as a_2, and the acceleration of block 1 as a_1. Two cases are possible:

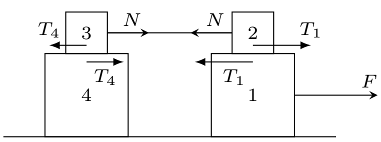

Fig. 4

1) No slipping between blocks 1 and 2, the system moves as a single body:

$$a_1 = a_2 = F/2(M+m).$$

The equation of motion for block 1: $Ma_1 = F - T_1$ and the condition for no slipping $T_1 \leqslant \mu mg$ allow us to find the condition that the force must satisfy for case one to hold: $F \leqslant 2\mu mg(M+m)/(M+2m)$.

2) Block 2 slides on block 1:
From the equation of motion for block 1: $a_1 = (F - \mu mg)/M$.
From the equation of motion for blocks 2, 3, and 4: $a_2 = \mu mg/(2m+M)$.
Case two occurs if the force satisfies the condition:
$F > 2\mu mg(M+m)/(M+2m)$.

789. The problem can be solved in a reference frame attached to the surface or the pencil.

1) Inertial frame attached to the surface: The moving pencil experiences two forces from the surface: the normal force N and the friction force T. The pencil cannot move vertically, so the normal force equals the gravitational force $N = mg$, and the friction force is $T = \mu N = \mu mg$. Consider the limiting case when the pencil begins to rotate around edge A (Fig. 5). The forces N and T are applied at edge A. If their resultant force R passes below the pencil's axis, its moment about the axis causes rotation. The condition for no rotation is:

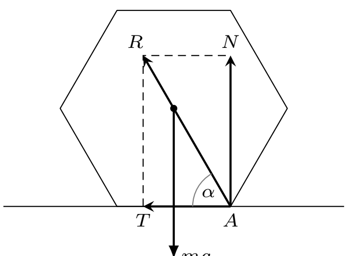

Fig. 5

$$\operatorname{tg}\alpha = N/T \geqslant \operatorname{tg} 60^\circ, \quad \mu \leqslant 1/\sqrt{3}.$$

2) Non-inertial frame attached to the pencil: In this frame, the pencil experiences an additional fictitious force due to acceleration caused by friction (Fig. 6): $F_b = ma = m\mu g$. The pencil rotates around edge A (instantaneous axis of rotation) if the moment of force F_b about A is greater than the moment of the gravitational force:

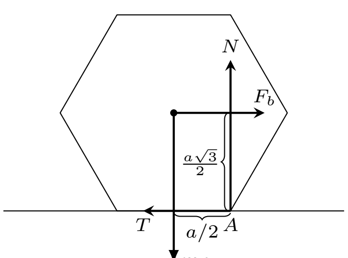

Fig. 6

$$F_b a\sqrt{3}/2 > mga/2.$$

From this, we obtain the same condition for no rotation as before.

Leaderboard of the problem-solving league **Klub 44 F** after considering the evaluations of solutions for problems 782 ($WT = 1{,}68$), 783 (WT=3,45) from issue 9/2024

Konrad Kapcia (Poznań)	3 – 44+3,25
Paweł Perkowski (Ożarów Maz.)	6 – 44+0,70
Jacek Konieczny (Poznań)	40,87
Tomasz Wietecha (Tarnów)	17 – 39,17
Jan Zambrzycki (Białystok)	4 – 28,35
Andrzej Nowogrodzki (Chocianów)	3 – 27,49

Club 44 M

Submission deadline: June 30, 2025

Top standings in the problem-solving league **Club 44 M** after including scores from problems 885 ($WT = 1{,}95$) and 886 ($WT = 1{,}42$) from issue 9/2024

Mikołaj Pater		42.95
Witold Bednarek	Łódź	42.00
Tomasz Wietecha	Tarnów	41.98
Krzysztof Zygan	Lubin	39.38
Andrzej Daniluk	Warsaw	37.89
Andrzej Kurach	Ryjewo	36.18
Michał Warmuz	Żywiec	33.21
Marcin Kasperski	Warsaw	32.31
Jędrzej Biedrzycki		32.29
Krzysztof Kamiński	Pabianice	31.90
Grzegorz Wiączkowski		31.79
Marian Łupieżowiec	Gliwice	31.29

Correction of an editorial error: In the annual summary of the participant list (*Delta* 2/2025), the balance for Grzegorz Wiączkowski should be listed as 29.40.

Mathematics Problems No. 899, 900

Edited by Marcin E. KUCZMA

899. Let $g\colon \mathbb{Z}^2 \to \mathbb{R}$ be an arbitrary function ($\mathbb{Z}^2$ is the set of lattice points, i.e., ordered pairs of integers). The distance between points $P, Q \in \mathbb{Z}^2$, where $P = (x, y)$ and $Q = (u, v)$, is defined as $d(P, Q) = \max(|x-u|, |y-v|)$. Prove that there exist infinitely many five-element sets of lattice points $(P_0, P_1, P_2, P_3, P_4)$ such that (for $i = 1, 2, 3, 4$): $d(P_0, P_i) = 1$, $g(P_0) \leqslant g(P_i)$.

900. Polynomials f_1, f_2, f_3, f_4 of one variable, with real coefficients, satisfy the conditions:

$$f_1(x) \leqslant f_2(x) \leqslant f_3(x) \leqslant f_4(x) \quad \text{for } x \in [0,1],$$
$$f_2(x) \leqslant f_4(x) \leqslant f_1(x) \leqslant f_3(x) \quad \text{for } x \in [-1,0].$$

Prove that $f_1 = f_2 = f_3 = f_4$.

Problem 900 was proposed by Mr. Michał Adamaszek from Copenhagen.

Solutions to problems from issue 12/2024

Reminder of the problems:

891. Find all real numbers φ that satisfy the condition: $\cos\bigl(2^n\varphi\bigr) \leqslant 0$ for every integer $n \geqslant 0$.

892. A badminton tournament involves $n \geqslant 2$ players, where each player competes against every other player once, with no draws. For each $k \in \{0, \ldots, n-1\}$, determine the maximum possible number of players who finished the tournament with exactly k wins.

891. Let $\cos\bigl(2^n\varphi\bigr) = x_n$ (so $|x_n| \leqslant 1$). From the cosine doubling formula, we obtain the recurrence relation $x_{n+1} = 2x_n^2 - 1$. We investigate for which initial values x_0 the condition: all $x_n \in [-1, 0]$ is satisfied.

Let $y_n = x_n + \frac{1}{2}$. The problem condition takes the form $|y_n| \leqslant \frac{1}{2}$, and the recurrence relation transforms into:

(1) $\qquad y_{n+1} = -y_n(2 - 2y_n) \quad$ for $n = 0, 1, 2, \ldots$

Since $|y_n| \leqslant \frac{1}{2}$, the expression in parentheses has a value $\geqslant 1$; and when $y_n \leqslant 0$, its value is $\geqslant 2$. Thus:

(2) $$|y_{n+1}| \geqslant \begin{cases} |y_n| & \text{for all } n, \\ 2|y_n| & \text{when } y_n \leqslant 0. \end{cases}$$

From equation (1), it follows that in every pair of consecutive terms of the sequence, there is a non-positive number. From property (2), we now deduce that $|y_{n+2}| \geqslant 2|y_n|$ for all n. By induction, we obtain $|y_{2n}| \geqslant 2^n|y_0|$; and since $|y_{2n}| \leqslant \frac{1}{2}$, we must have $y_0 = 0$. This corresponds to the value $x_0 = -\frac{1}{2}$ (and then all $x_n = -\frac{1}{2}$).

The required condition is therefore satisfied if and only if $x_n = \cos\varphi = -\frac{1}{2}$, which holds only for $\varphi = \pm\frac{2}{3}\pi + 2m\pi$ ($m = 0, \pm1, \pm2, \ldots$).

892. Solution by the author (Michał Adamaszek). Assume that a player earns 1 point for a win and 0 for a loss. Fix k and let S be the set of players who finished the tournament with k points; let $|S| = s$. Within S, they played $\binom{s}{2}$ matches, so they collectively earned at least that many points. However, by definition of S, they earned exactly ks points in total. Thus, $\binom{s}{2} \leqslant ks$, meaning $s \leqslant 2k+1$.

Now assume that all match results in the tournament were reversed ($0 \mapsto 1$, $1 \mapsto 0$). In this new tournament, S is the set of players who earned exactly $n-k-1$ points, so the same reasoning gives that $\binom{s}{2} \leqslant (n-k-1)s$, meaning $s \leqslant 2n-2k-1$. Therefore,

$$s \leqslant \min\{2k+1, 2n-2k-1\}.$$

We show that this bound is achievable. In the case when $1 \leqslant 2k+1 \leqslant n$, let S be any set of players of that exact size. It is easy to arrange matches within S so that each player wins exactly k matches (e.g., each player beats k of their successors in any cyclic arrangement of S). Players outside S win all their matches against S and compete arbitrarily among themselves. Then each player in S earns k points, while each player outside S earns at least $2k+1 \geqslant k+1$ points.

In the case when $n+1 \leqslant 2k+1 \leqslant 2n-1$, let $j = n-1-k$. Then $1 \leqslant 2j+1 \leqslant n-1$, so by the previous construction, there exists a tournament in which exactly $2j+1 = 2n-2k-1$ players have scored exactly j points. By reversing all results to their opposites, we obtain a tournament in which exactly $2n-2k-1$ players have scored exactly k points.

Thus, the desired maximum value of $|S|$ is $\min\{2k+1, 2n-2k-1\}$.

Skrót regulaminu

Anyone may submit solutions to problems from issue n by the end of month $n+2$. Solution sketches are published in issue $n+4$. One may submit solutions to four, three, two, or one problem (each on a separate sheet), either every month or with any breaks. Solutions to mathematics and physics problems should be sent in separate envelopes, with the label: **Club 44 M** or **Club 44 F** on the envelope. They can also be sent via email to `delta@mimuw.edu.pl` (we prefer PDF files). The problems are evaluated on a scale from 0 to 1, with an accuracy of 0.1. The score is multiplied by the difficulty factor of the problem: $WT = 4 - \frac{3S}{N}$, where S denotes the sum of scores for solutions to this problem, and N is the number of people who submitted solutions to at least one problem from the given issue in the given category (**M** or **F**)—this is the number of points awarded to the participant. Upon accumulating **44** points, at any time and in either of the two categories (**M** or **F**), the participant becomes a member of **Club 44**, and any excess points are carried over for future participation. Achieving membership three times grants the title of **Veteran**. The detailed rules were published in issue 2/2002 and can also be found on the website `deltami.edu.pl`.