Beyond Classical Limits: The Power and Potential of Quantum Computing

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Imagine a computer that could break codes that would usually take millions of years to crack in seconds. Imagine a computer that could process multiple possibilities simultaneously. Quantum computing makes this possible by leveraging the laws of quantum mechanics—the fundamental rules that govern the behavior of the smallest particles—and manipulating their strange properties to increase computational speed to levels incomprehensible for classical computers.

The world's most common encryption algorithm, RSA encryption, works on the principle that it is difficult to factorize large numbers. However, Shor's algorithm, a quantum algorithm named after Peter Shor, can factorize integers in $O(\log(N)^2)$ time.

Two key quantum properties that I will explore are *superposition* and *entanglement*.

Superposition

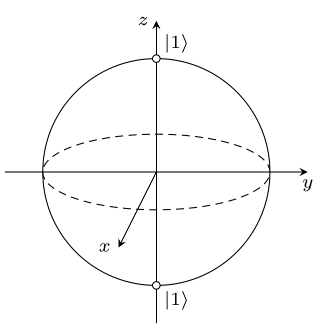
While classical computers rely on bits that exist in a state of 0 or 1, quantum computers use qubits (quantum bits), which can exist in a state of 0, 1, or both simultaneously, thanks to the phenomenon of superposition. To understand this concept, we can explore the mathematics behind it. A quantum state can be written as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

Where α and β are complex numbers, normalized such that $|\alpha|^2 + |\beta|^2 = 1$. Here, $|\alpha|^2$ and $|\beta|^2$ correspond to the probabilities of the qubit being in the $|0\rangle$ and $|1\rangle$ states, respectively.

For example, the state $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ corresponds to the qubit being equally likely to be either in the $|0\rangle$ or $|1\rangle$ state when measured.

All possible states of a qubit can be represented by points on a sphere in three-dimensional space. The poles of the sphere represent the $|1\rangle$ and $|0\rangle$ states, and the *equator* represents states that are in perfect superposition. This representation, known as the Bloch sphere, is named after physicist Felix Bloch.



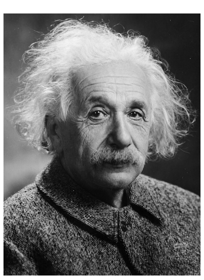
The concept of superposition is fundamental to quantum computing, as it allows for parallel processing. Since one qubit can hold information about two states at once, two qubits in superposition can represent four states ($|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$), and n qubits can represent 2^n states simultaneously. This allows for more efficient search algorithms, such as Grover's algorithm, which can search through N unsorted items in $\sqrt{N}$ searches on average, breaking the classical computing limit of N searches for a list of unsorted items.

Grover's algorithm works using a process known as *amplitude amplification*. Imagine we have a set of 32 items, with one marked as the correct item. Initially, a superposition of 5 qubits is set up, with each of the 32 output states corresponding to one of the items. Then, a set of quantum operators is applied to increase the probability of the correct item being chosen. This process is repeated until all 5 qubits reflect either the $|1\rangle$ or $|0\rangle$ state, corresponding to the correct item.

Entanglement

Quantum entanglement is the property of quantum particles to become linked. When two quantum particles are entangled, information about one particle provides information about the other. A good analogy is a pair of shoes. Imagine you take each shoe from a pair and place them in separate boxes. By checking one box, you can tell whether the shoe is left or right, but you also know whether the other shoe in the other box, which you haven't opened, is left or right. You gain information about the other shoe without opening the box. The difference is that shoes cannot be in superposition of different states, but qubits can.

The concept of particles having an effect on each other from a distance is counterintuitive, even to Albert Einstein, who referred to quantum entanglement as "spooky action at a distance."



For example, consider a state of two qubits, which is a superposition of states $|00\rangle$ and $|11\rangle$ with equal probabilities. So, we know that both qubits are in the same state, but we do not know if it is 0 or 1. This state is described by:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

It turns out that this state is entangled. If we measure the first qubit and find it in the state $|0\rangle$, then we can be sure that the second qubit will also be in $|0\rangle$. Similarly, if the first qubit is found in $|1\rangle$, the second qubit will also be in $|1\rangle$. This correlation holds regardless of the distance between the qubits and enables a level of coordination that classical bits cannot achieve. This means the probability of observing the states $|01\rangle$ or $|10\rangle$ is 0.

In 2012, researchers achieved quantum teleportation over a distance of 143 km, from La Palma to Tenerife *arXiv:1205.3909 [quant-ph].*

Applications of entanglement are widespread in quantum algorithms. For example, quantum entanglement enables superdense coding, a quantum algorithm that aims to transmit a number of classical bits of information using fewer qubits. Another application of entanglement is quantum teleportation. This theoretical process allows for information transfer through entangled particles. In this process, two parties—located at any distance from each other—use a shared entangled state to transfer information about a given quantum state from one location to another. This process is referred to as the teleportation of a quantum state.

Current Quantum Technology

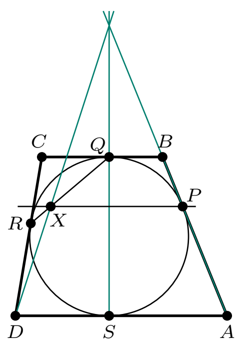
The world's largest quantum computer is a 1,180-qubit system developed by Atom Computing. Each qubit is a neutral atom that is trapped and controlled by an array of lasers.

Quantum technology is still in its early stages, but there has been significant progress. Large tech companies like Google, IBM, and Intel are building computers with increasingly more qubits; however, they are struggling to reduce error rates. Currently, we have noisy intermediate-scale quantum (NISQ) devices, which have enough qubits to perform certain quantum computations but are prone to errors and decoherence (the loss of quantum information). Scientists are working on developing error-correction techniques and „quantum error-correcting codes" to reduce decoherence. The ultimate goal is to build error-resistant quantum computers with applications across various industries. Large-scale quantum computers may still be a decade away, but the current pace of innovation suggests a bright future for quantum technology.

Problems

Edited by Dominik BUREK

M 1813. Let $ABCD$ be a trapezoid ($DA \parallel CB$) circumscribed on a circle that is tangent to the sides AB, BC, CD, and AD at points P, Q, R, and S, respectively. A line passing through P and parallel to the bases of the trapezoid intersects the line QR at point X. Prove that the lines AB, QS, and DX intersect at a single point.



M 1814. Given numbers a, $b > 1$, for which
$$a + \frac{1}{a^2} \geqslant 5b - \frac{3}{b^2}.$$
Prove that
$$a > 5b - \frac{4}{b^2}.$$

M 1815. Consider two integers $n > 20$ and $k > 1$ such that $k^2 | n$. Prove that there exist positive integers a, b, and c such that
$$n = ab + bc + ca.$$

Edited by Andrzej MAJHOFER

F 1117. In a tightly sealed cylinder, under the piston, there are $m = 10$ g of liquid water. A very rapid movement of the piston causes the pressure in the cylinder to drop to nearly zero. The surrounding temperature and the cylinder with water are at $0°C$. How much ice will be produced as a result of this process? It can be assumed that initially, under the piston, there was only liquid water. The latent heat of fusion of water is $L_f \approx 334$ J/g, and the latent heat of vaporization is $L_v \approx 2260$ J/g.

F 1118. In a sealed container, there is a mixture of helium and neon. The mixture is in thermodynamic equilibrium, with the number of moles of neon and helium being the same. A very small hole is made in the wall of the container. What will be the composition of the gas beam escaping from the container just after the hole is made? In atomic mass units, the atomic masses are: helium $\mu_{\mathrm{He}} = 4$, and neon $\mu_{\mathrm{Ne}} = 20$.

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